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13. ABSTRACT (Maximum 200 words)

An effective computational strategy is developed for generating the response of complex systems using (small or large) perturbations from the response of a simple structure (or a simpler mathematical/discrete model of the original structure). Two general approaches are developed for selecting the simpler model and establishing the relations between the original and simpler models. The two approaches are: decomposition or partitioning strategy, and hierarchical modeling strategy. Two effective partitioning strategies are used. The first is based on uncoupling of load-carrying mechanisms, and the second is based on symmetry transformations. The hierarchical modeling used is a predictor-corrector iterational process based on using a simple mathematical model in the predictor phase and correcting the response using a more accurate mathematical model. The strategies have been applied to several problems including: thermal buckling and postbuckling of multilayered composite plates; and nonlinear dynamic analysis of composite shells.

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TABLE OF CONTENTS

ABSTRACT

An effective computational strategy is developed for generating the response of complex systems using (small or large) perturbations from the response of a simple structure (or a simpler mathematical/discrete model of the original structure). Two general approaches are developed for selecting the simpler model and establishing the relations between the original and simpler models. The two approaches are: decomposition or partitioning strategy, and hierarchical modeling strategy. Two effective partitioning strategies are used. The first is based on uncoupling of load-carrying mechanisms, and the second is based on symmetry transformations. The hierarchical modeling used is a predictor-corrector iterational process based on using a simple mathematical model in the predictor phase and correcting the response using a more accurate mathematical model. The strategies have been applied to several problems including: thermal buckling and postbuckling of multilayered composite plates; and nonlinear dynamic analysis of composite shells.

RESEARCH OBJECTIVES

The objective of the present study is to develop an effective computational method for generating the response of a complex system using large perturbations from that of a lower-order model associated with a simpler system (or a simpler mathematical/discrete model of the original system). As an integral part of the proposed strategy an attempt will be made to unify and realize the full potential of a number of multilevel computational strategies, some of which were developed by the principal investigator and his colleagues. The multilevel strategies include reduction methods, hybrid modeling/analysis techniques, and partitioning methods. Reduction methods are techniques for substantially reducing the number of degrees of freedom of the initial discretization, and have been successfully applied to a number of vibration and nonlinear structural and thermal problems. Hybrid modeling/analysis techniques can achieve significant reductions in the analysis time by incorporating the known physical behavior into the computational model of the system and by using different analysis methods and/or models in predicting the different response characteristics of the engineering systems. Partitioning methods are based on breaking the large (and/or complex) problem into a number of smaller (and/or simpler) subproblems. The solution of the original problem is generated using information provided by the individual subproblems.

The proposed strategy is believed to combine the following three major characteristics:

1) gives physical insight about the response

2) helps in assessing the adequacy of the computational model; and

3) is highly efficient.

The strategy will first be applied to: a) the nonlinear postbuckling problem of composite structures; b) reanalysis of large structures in the presence of geometric nonlinearities; then c) coupled field problems. The postbuckling response of the highly anisotropic composite structure is generated using large perturbations from the response of a simpler structure. The three key elements of the strategy to be exploited in the first two applications are: 1) mixed (or primitive variable) formulation, with the fundamental unknowns consisting of both stress and displacement parameters; 2) operator splitting, or additive decomposition of the different arrays in the equations of the given structure to the corresponding arrays of the simpler, or previously-analyzed, structure plus correction terms; and c) application of a reduction method and/or a stable iterative method for the efficient generation of the equations of the given structure.

RESEARCH ACCOMPLISHMENTS

The results of the research conducted during the period September 30, 1990 to August 31, 1993 are included in thirteen publications. Three of the tasks that have been performed under this grant, namely, the development of an improved partitioning strategy for large-scale structural problems; the development of a predictor-corrector approach for generating the steady-state thermal response and thermal buckling of multilayered composite plates and shells; and hierarchical adaptive modeling of structural sandwiches, are described subsequently. An overview of recent activities and a summary of the accomplishments were presented at the Air Force Contractors Meeting in Dayton, Ohio, October 10-11, 1991 (see Appendix I). The abstracts of the papers completed under the grant are given in Appendix II.

Improved Partitioning Strategy for Large-Scale Problems

The governing equations for the discrete model of the original structure can be written in the following compact form:

$$[K]\{Z\} = \{Q\} \tag{1}$$

where $\{Z\}$ is the vector of stress parameters and generalized displacements; $[K]$ is the global structure matrix which includes the flexibility and strain-displacement matrices; and $\{Q\}$ is the global right-hand-side vector.

In the decomposition strategy the vector of fundamental unknowns $\{Z\}$ is partitioned into smaller subvectors. The governing discrete equations, Eqs. 1, are partitioned accordingly. The simpler model is associated with the uncoupled equations in the partitioned variables. For the case of two partitions, the process can be described by embedding Eqs. 1 in a single-parameter family of equations as follows:

$$\left(\begin{bmatrix} K_{11} & \cdot \\ \cdot & K_{22} \end{bmatrix} + \lambda \begin{bmatrix} \cdot & K_{12} \\ K_{21} & \cdot \end{bmatrix}\right)\begin{Bmatrix} Z_1 \\ Z_2 \end{Bmatrix} = \begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} \tag{2}$$

where $\{Z_1\}$, $\{Z_2\}$ and $\{Q_1\}$, $\{Q_2\}$ are the partitions of the original vectors $\{Z\}$ and $\{Q\}$; λ is a tracing parameter which identifies all the correction terms needed in going from the simpler model to the discrete model of the original structure; and a dot (.) refers to a zero submatrix. The simpler model corresponds to $\lambda=0$ (uncoupled equations in $\{Z_1\}$ amd $\{Z_2\}$), and the discrete model of the original structure corresponds to $\lambda=1$ (fully coupled equations). The solution corresponding to $\lambda=1$ is generated from the corresponding solution at $\lambda=0$ using an iterative process such as the Preconditioned Conjugate Gradient (PCG). Note that the correction vectors of the iterative process, provide a direct measure of the sensitivity of the response quantities to the coupling terms (viz., the terms associated with the tracing parameter λ in Eq. 2).

The vectors $\{Z_1\}$ amd $\{Z_2\}$ are chosen to be the symmetric and antisymmetric components of the response vector (each is approximately half the size of the original vector, $\{Z\}$). The simpler model ($\lambda=0$) corresponds to a symmetrized structure in which the symmetric and antisymmetric components of the response vector are uncoupled. This approach can be thought of as a physical domain decomposition. If the PCG technique is used in generating the solution at $\lambda=1$, and the preconditioning matrix is selected to be the left-hand side matrix corresponding to $\lambda=0$, then each of the correction vectors is either symmetric or antisymmetric.

The convergence of the PCG technique can be expedited by replacing Eqs. 2 by the following equivalent form of two uncoupled equations in $\{Z_1\}$ amd $\{Z_2\}$:

$$\left(\begin{bmatrix} K_{11} & \cdot \\ \cdot & K_{22} \end{bmatrix} - \lambda \begin{bmatrix} K_{12}K_{22}^{-1}K_{21} & \cdot \\ \cdot & K_{21}K_{11}^{-1}K_{12} \end{bmatrix}\right)\begin{Bmatrix} Z_1 \\ Z_2 \end{Bmatrix} = \begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} - \lambda \begin{Bmatrix} K_{12}K_{22}^{-1}Q_2 \\ K_{21}K_{11}^{-1}Q_1 \end{Bmatrix} \tag{3}$$

Note that for $\lambda=1$, each diagonal block of the total left-hand-side matrix is in the form of Schur complement which is not formed explicitly. Rather, the preconditioning matrix is selected to be the first matrix on the left-hand side (corresponding to $\lambda=0$) and the PCG technique is used in generating the solution at $\lambda=1$. The results of this research are reported in Ref. 1.

Predictor-Corrector Approach for Generating the Steady-State Temperature Distribution and Thermal Buckling Response of Multilayered Plates and Cylinders

A predictor-corrector procedure has been developed for the accurate determination of the temperature and heat flux distributions, as well as the thermal buckling response of thick multilayered composite plates and shells. The procedure is based on using a linear through-the-thickness temperature (or displacement) distribution in the predictor phase. The functional dependence of temperature (or displacements) on the thickness coordinate is then calculated a posteriori and used in the corrector phase.

Extensive numerical results have been conducted for linear steady-state heat conduction as well as thermal buckling problems, showing the effects of variation in the geometric and lamination parameters on the accuracy of the temperature distribution and thermal buckling response predictions of the predictor-corrector approach. Both antisymmetrically laminated anisotropic plates, and multilayered orthotropic cylinders are considered. The solutions are assumed to be periodic in the surface coordinates. For each problem the standard of comparison is taken to be the analytic three-dimensional solution based on treating each layer as a homogeneous anisotropic medium. The potential of the predictor-corrector approach for predicting the thermal response of multilayered plates and shells with complicated geometry is investigated. The results of this study have been reported in publications.

Hierarchical Adaptive Modeling of Structural Sandwiches

The key elements of a hierarchical adaptive modeling strategy for structural sandwiches have been identified. The strategy uses the multimodel predictor-corrector modeling procedures. Some initial results have been obtained for free vibrations of thermally stressed multilayered composite panels and structural sandwiches with composite face sheets. The results have been accepted for publication (Ref. 16). The strategy appears to have high potential for more complex problems.

PUBLICATIONS

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14. Noor, A. K., "Mechanics of Anisotropic Plates and Shells - A New Look at an Old Subject," in Developments in Theoretical and Applied Mechanics, in Developments in Theoretical and Applied Mechanics, Proceedings of the SECTAM XVI Conference, edited by B. Antar, R. Engels, A. A. Prinaris and T. H. Moulden, The University of Tennessee Space Institute, Tullahoma, TN, April 1992, pp. I.2.1-I.2.24; also Computers and Structures, Vol. 44, No. 3, 1992, pp. 499-514.

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18. Noor, A. K. and Peters, J. M., "Finite Element Buckling and Postbuckling Solutions for Multilayered Composite Panels," Finite Elements in Analysis and Design (to appear).

19. Noor, A. K., Kim, Y. H. and Peters, J. M., "Transverse Shear Stresses and Their Sensitivity Coefficients in Multilayered Composite Panels," AIAA Journal (to appear).

EFFECTIVE COMPUTATIONAL STRATEGIES FOR PREDICTING THE RESPONSE OF COMPLEX SYSTEMS
AFOSR Grant No. 90-0369

Principal Investigator: Ahmed K. Noor

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OVERVIEW:
The present research aims at developing effective computational procedures for generating the response of a complex system using *large perturbations* from that of a lower-order model associated with a simpler system (or a simpler mathematical/discrete model of the original system). The strategies developed will allow the practical use of more sophisticated models, for the accurate prediction of the response of future flight vehicles, than has heretofore been done. This is accomplished, among other things, by: a) strongly coupling the physics of the problem with the solution strategy, and b) attempting to unify a number of multilevel computational strategies, including reduction methods, hybrid modeling/analysis techniques, and partitioning methods.

SUMMARY OF ACCOMPLISHMENTS:
The computational strategies, which have been developed for predicting the response of complex systems, combine the following three major characteristics:

1. *Provide physical insight about the response.* This is accomplished by using hierarchical adaptive modeling, in the sense of starting from a simpler model and increasing the level of sophistication, as needed, to model the actual engineering system.

2. *Help in assessing the adequacy of the computational model.* This is accomplished by obtaining sensitivity information about the modeling details neglected, as part of the analysis process.

3. *High computational efficiency.* This is achieved by: a) reducing the number of degrees of freedom used in the initial discretization; and b) exploiting the major features of new computing systems (viz., vector, parallel and AI capabilities).

The basic idea of the strategies developed is to generate the response of the complex system using *large perturbations* from that of a simpler model associated with either a simpler system or a simpler mathematical/discrete model of the original system.

The discrete equations of the simpler system are embedded into those of the original complex system. Sensitivity derivatives of the response of the system with respect to the modeling details neglected, are directly available.

The response of the simpler system (or simpler model) is then used as a predictor and a stable iterative process (e.g., PCG or multigrid technique) is applied to generate the response of the original system.

Two general approaches have been developed for selecting the simpler model, namely, a) hierarchical modeling (multimodel or multigrid), and b) decomposition or partitioning (physical domain decomposition). The *first approach* is based on using either a mathematical model of a lower dimensionality (multimodel or physical multigrid - PMG); or a coarse finite element grid (classical multigrid). The *second approach* is based on either uncoupling of different fields in coupled problems; uncoupling of load carrying mechanisms in structural problems; or using symmetry transformations. Hierarchical modeling strategies (predictor-corrector approaches) have been developed for the accurate prediction of the response characteristics of multilayered composite plates and shells (see Refs. 1 and 2). The basic idea of these strategies is summaized in Fig. 1.

The strategies developed have been applied to several problems including: thermal buckling and postbuckling of multilayered composite plates; and nonlinear dynamic analysis of composite shells. Typical results are shown in Figs. 2, 3 and 4. Figure 2 shows the accuracy of the critical temperature and its derivative with respect to Young's modulus in the fiber direction obtained by the predictor-corrector approaches described in Refs. 1 and 2. The plates considered are ten-layered angle-ply composites subjected to uniform temperatures. The standard of comparison is taken to be the exact three-dimensional thermoelasticity solution (Refs. 3 and 4). The results obtained by the predictor-corrector procedures are compared with those obtained by other modeling approaches based on different through-the-thickness approximations for the displacements v_1, v_2, ω and the transverse stresses τ_{13}, τ_{23} and τ_{33} (see Fig. 2). Figure 3 shows the accuracy of the transverse shear stresses obtained by the different modeling approaches. The effectiveness of the predictor-corrector approaches is clearly demonstrated in Figs. 2 and 3.

Figure 4 shows a cylindrical panel with an off-center cutout subjected to uniform normal loading. The decomposition strategy based on symmetry transformations was applied to the evaluation of the transient dynamic response of the panel. The performance of the strategy on CRAY YMP4/432 computer is shown in Table 1. As can be seen from Table 2, the speedup resulting from the use of the strategy is over one order of magnitude.

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1. Noor, A. K. and Burton, W. S., "Predictor-Corrector Procedures for Thermal Buckling of Multilayered Composite Plates," Computers and Structures, Vol. 40, No. 5, 1991, pp. 1071-1084.
2. Noor, A. K. and Burton, W. S., "Accuracy of the Critical-Temperature Sensitivity Derivatives Predicted by Two-Dimensional Multilayered Composite Plate Theories," AIAA Journal (to appear).
3. Noor, A. K. and Burton, W. S., "Three-Dimensional Solutions for Thermal Buckling of Multilayered Anisotropic Plates," Journal of Engineering Mechanics, ASCE (to appear).
4. Noor, A. K. and Burton, W. S., "Three-Dimensional Solutions for the Free Vibrations and Buckling of Thermally Stressed Multilayered Composite Plates," Journal of Applied Mechanics (to appear).
5. Noor, A. K. and Peters, J. M., "Strategies for Large-Scale Structural Problems on High-Performance Computers," Communications in Applied Numerical Computers (to appear).

Predictor Phase

- Use first-order shear deformation theory to predict global response characteristics, in-plane strain and stress components
- Use three-dimensional equilibrium equations to calculate transverse stresses and strains

Corrector Phase

- Adjust (calculate *a posteriori*) some key elements of the computational model
  - Composite shear correction factors (Approach I)

  or
  - Functional dependence of displacement components on the thickness coordinate (Approach II)
- Use the adjusted elements of the model to improve the accuracy of the various response quantities

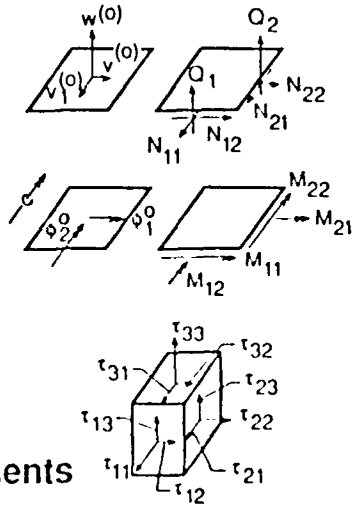


Figure 1 - Basic idea of the predictor-corrector approaches used in the present study.

Model no.	v_1, v_2	ω	τ_{33}	$\tau_{13,23}$	
1	Linear	Const.	0	$k_1 = k_2 = 1$	○
1A	Linear	Const.	0	$k_1 = k_2 = 5/6$	□
2	Linear	Linear	-	-	◇
3	Cubic	Quadratic	-	-	◺
4	Quintic	Quintic	-	-	D
5	Cubic	Const.	-	0 at h/2	△
6, 6A	Predictor-corrector procedures				◇ ◊ ⌂

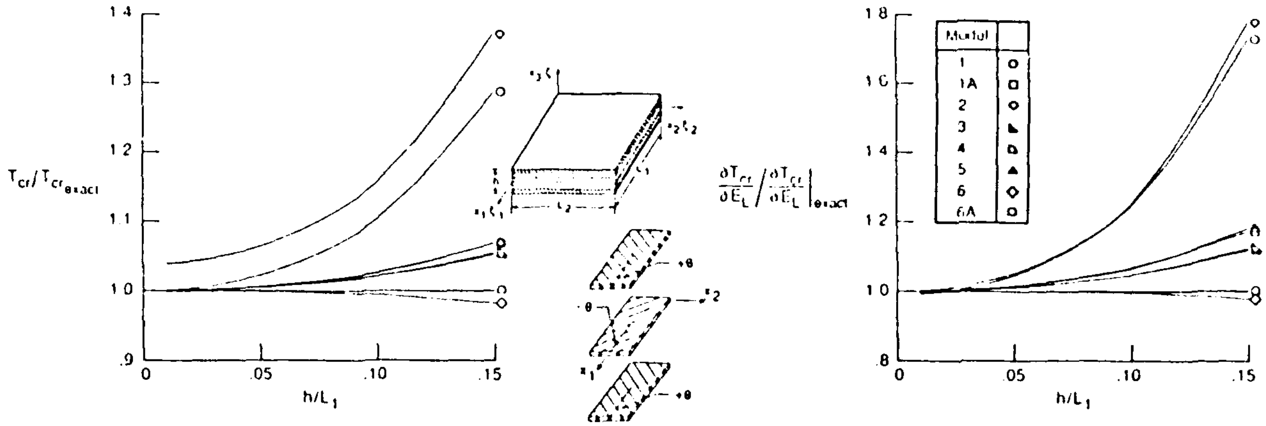


Figure 2 - Accuracy of critical temperature and its sensitivity derivative obtained by the predictor-corrector and other modeling approaches. Ten-layered angle-ply plate with θ=45°.

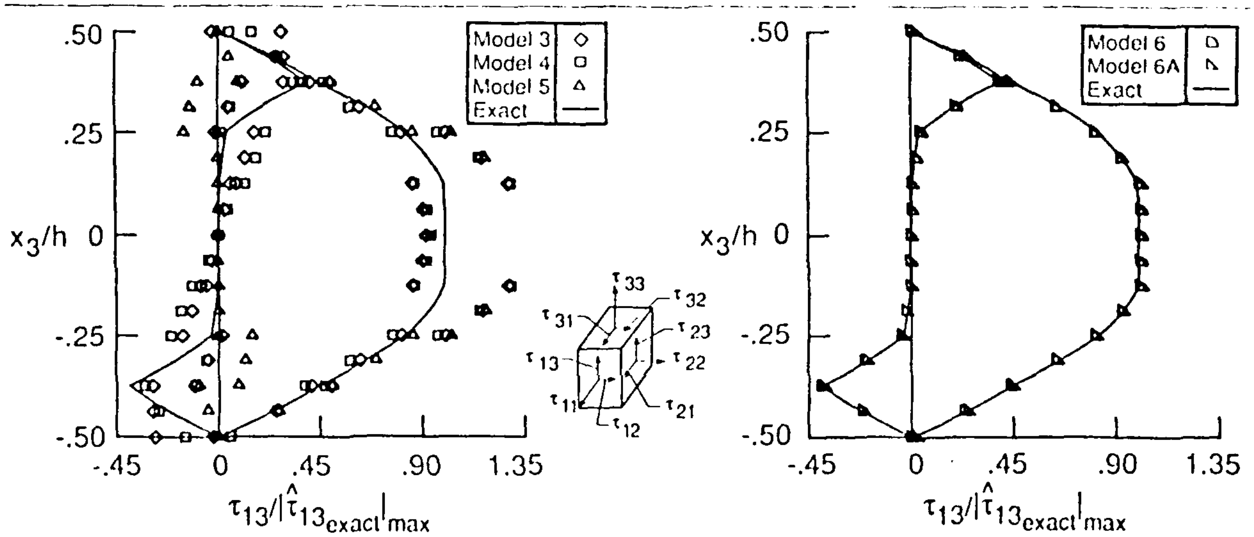
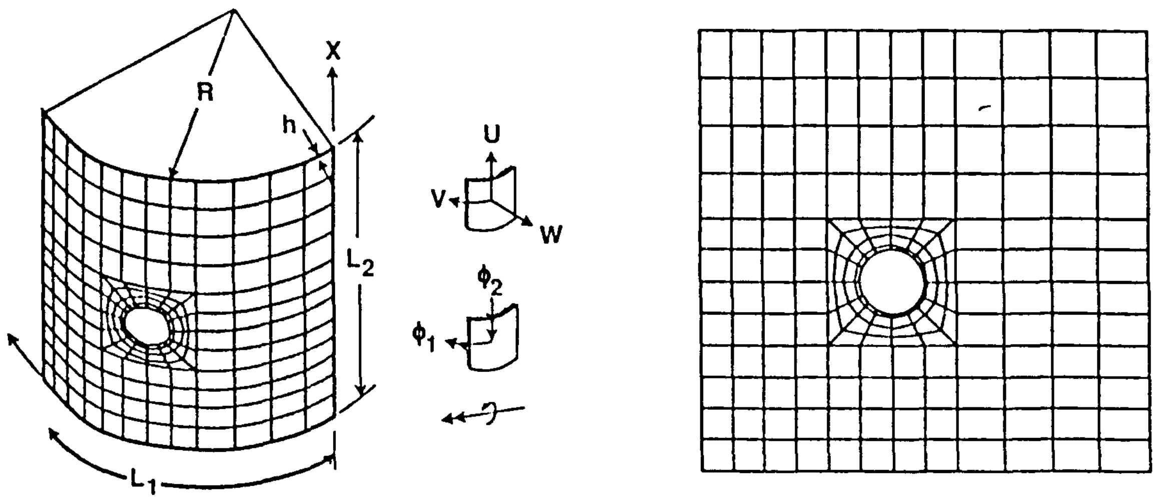


Figure 3 - Accuracy of transverse shear stresses τ_{13} obtained by the predictor-corrector and other modeling approaches (see Fig. 2).



3818 nonzero displacement degrees of freedom
6144 stress degrees of freedom

Loading

Uniform normal loading with intensity p_o

Boundary conditions

At $x = 0, L_1$:

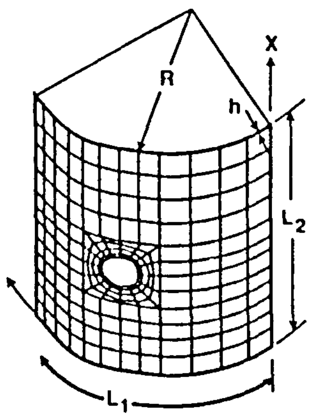
$$u = v = w = \phi_1 = \phi_2 = 0$$

At $y = 0, L_2$:

$$w = \phi_1 = 0$$

Figure 4 - Cylindrical panel with an off-center cut-out used in the present study.

	Full Structure (Optimized Code) (278 MFLOPS)	Partitioned Structure (Nearly Optimized Code) (246 MFLOPS)
Number of degrees of freedom	3818 displacements 6144 stress	971 displacements 1536 stresses
Semibandwidth of equations	700	315
Wall clock time (sec.) (first ten steps)	171	58.6 one processor 29.7 two processors 16.4 four processors
Speedup	1.0	2.92 one processor 5.76 two processors 10.43 four processors



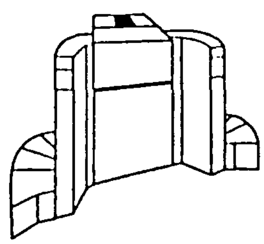


Table 1 - Performance evaluation of the decomposition strategy based on symmetry transformations on the CRAY-Y MP4/432 at Mendota Heights.

APPENDIX II

Abstracts of Papers Completed Under the Grant

COMMUNICATIONS IN APPLIED NUMERICAL METHODS, Vol. 7, 465–478 (1991)

STRATEGIES FOR LARGE SCALE STRUCTURAL PROBLEMS ON HIGH-PERFORMANCE COMPUTERS

AHMED K. NOOR AND JEANNE M. PETERS

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SUMMARY

Novel computational strategies are presented for the analysis of large and complex structures. The strategies are based on generating the response of the complex structure using large perturbations from that of a simpler model, associated with a simpler structure (or a simpler mathematical/discrete model of the original structure). Numerical examples are presented to demonstrate the effectiveness of the strategies developed.

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Printed in Great Britain.

0045-7949/91 $3.00 + 0.00
Pergamon Press plc

STEADY-STATE HEAT CONDUCTION IN MULTILAYERED COMPOSITE PLATES AND SHELLS

A. K. Noor and W. S. Burton

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(*Received* 3 *October* 1990)

Abstract—A study is made of a predictor–corrector procedure for the accurate determination of the temperature and heat flux distributions in thick multilayered composite plates and shells. A linear through-the-thickness temperature distribution is used in the predictor phase. The functional dependence of temperature on the thickness coordinate is then calculated *a posteriori* and used in the corrector phase.

Extensive numerical results are presented for linear steady-state heat conduction problems, showing the effects of variation in the geometric and lamination parameters on the accuracy of the thermal response predictions of the predictor–corrector approach. Both antisymmetrically laminated anisotropic plates and multilayered orthotropic cylinders are considered. The solutions are assumed to be periodic in the surface coordinates. For each problem the standard of comparison is taken to be the analytic three-dimensional solution based on treating each layer as a homogeneous anisotropic medium. The potential of the predictor–corrector approach for predicting the thermal response of multilayered plates and shells with complicated geometry is discussed.

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Pergamon Press Ltd

HYBRID ANALYTICAL TECHNIQUE FOR THE NONLINEAR ANALYSIS OF CURVED BEAMS

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(Received 22 May 1991)

Abstract—The application of a two-step hybrid technique to the geometrically non‑ ‑ar analysis of curved beams is used to demonstrate the potential of hybrid analytical techniques in nonlinear structural mechanics. The hybrid technique is based on successive use of the perturbation method and a classical direct variational procedure. The functions associated with the various-order terms in the perturbation expansion of the fundamental unknowns, and their sensitivity derivatives with respect to material and geometric parameters of the beam, are first obtained by using the perturbation method. These functions are selected as coordinate functions (or modes) and the classical direct variational technique is then used to compute their amplitudes. The potential of the proposed hybrid technique for nonlinear analysis of structures is discussed. The effectiveness of the hybrid technique is demonstrated by means of numerical examples. The symbolic computation system Mathematica is used in the present study. The tasks performed on Mathematica include: (1) generation of algebraic expressions for the perturbation functions of the different response quantities and their sensitivity derivatives; and (2) determination of the radius of convergence of the perturbation series.

THREE-DIMENSIONAL SOLUTIONS FOR THERMAL BUCKLING OF MULTILAYERED ANISOTROPIC PLATES

By Ahmed K. Noor,[1] Fellow, ASCE, and W. Scott Burton[2]

ABSTRACT: Analytic three-dimensional elasticity solutions are presented for the thermal buckling problem of multilayered anisotropic plates. The plates are assumed to have rectangular geometry and an antisymmetric lamination with respect to the middle plane. The temperature is assumed to be independent of the surface coordinates, but it has an arbitrary symmetric variation through the thickness of the plate. No external loads are present, but the motion of the plate is partially restrained in its plane. A mixed formulation is used, with the fundamental unknowns consisting of the six stress components and the three displacement components of the plate. The prebuckling deformations are taken into account. Each of the plate variables is decomposed into symmetric and antisymmetric components in the thickness direction, and is expressed in terms of a double Fourier series in the Cartesian surface coordinates. Extensive numerical results are presented showing the effects of the prebuckling deformation on the critical temperature, as well as the effects of variation in the lamination and geometric parameters of composite plates on the importance of the transverse stress and strain components.

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COMPUTATIONAL TECHNOLOGY FOR HIGH-TEMPERATURE AEROSPACE STRUCTURES

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Abstract—The status and some recent developments of computational technology for high-temperature aerospace structures are summarized. Discussion focuses on a number of aspects including: goals of computational technology for high-temperature structures; computational material modeling; life prediction methodology; computational modeling of high-temperature composites; error estimation and adaptive improvement strategies; strategies for solution of fluid flow/thermal/structural problems; and probabilistic methods and stochastic modeling approaches, integrated analysis and design. Recent trends in high-performance computing environment are described and the research areas which have high potential for meeting future technological needs are identified.

Thermomechanical Buckling of Multilayered Composite Plates

By Ahmed K. Noor,[1] Fellow, ASCE, and Jeanne M. Peters[2]

Abstract: A study is made of the thermomechanical buckling of composite plates subjected to combined thermal and axial loadings. The plates considered consist of a number of perfectly bonded layers and have symmetric lamination with respect to the middle plane. The material properties are assumed to be independent of temperature. The analysis is based on a first-order shear deformation theory. A mixed formulation is used, with the fundamental unknowns consisting of the generalized displacements and the stress resultants of the plate. An efficient multiple-parameter reduction method is used, in conjunction with mixed finite-element models, for determining the stability boundary of the plate. Sensitivity derivatives are evaluated and used to study the sensitivity of the buckling response to variations in different lamination and material parameters of the plate. Numerical results are presented that show the effectiveness of the reduction method, as well as the effects of variations in the material characteristics and fiber orientation of individual layers on the stability boundary of the plate.

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PREDICTOR-CORRECTOR PROCEDURES FOR THERMAL BUCKLING ANALYSIS OF MULTILAYERED COMPOSITE PLATES

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Abstract—A study is made of two predictor-corrector procedures for the accurate determination of global, as well as detailed, thermal buckling response characteristics of composite plates. Both procedures use first-order shear deformation theory in the predictor phase, but differ in the elements of the computational model being adjusted in the corrector phase. The first procedure calculates *a posteriori* estimates of the composite correction factors and uses them to adjust the transverse shear stiffnesses of the plate. The second procedure calculates *a posteriori* the functional dependence of the displacement components on the thickness coordinate. The corrected quantities are then used in conjunction with three-dimensional equations to obtain better estimates for the different response quantities. Extensive numerical results are presented, showing the effects of variation in the geometric and lamination parameters for antisymmetrically laminated composite plates subjected to uniform temperature rise, on the accuracy of the thermal buckling response obtained by predictor-corrector procedures. Comparison is also made with solutions obtained by other computational models based on two-dimensional shear deformation theories. For each problem, the standard of comparison is taken to be the analytic three-dimensional thermoelasticity solution, with prebuckling deformations accounted for. The numerical examples clearly demonstrate the accuracy and effectiveness of the predictor-corrector procedures.

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Postbuckling of multilayered composite plates subjected to combined axial and thermal loads

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Revised January 1992

Abstract. A study is made of the postbuckling response of composite plates subjected to combined axial and thermal loadings. The analysis is based on a first-order shear deformation, von Kármán-type nonlinear plate theory. A mixed formulation is used with the fundamental unknowns consisting of the generalized displacements and the stress resultants of the plate. An efficient reduction method is used in conjunction with mixed finite element models for determining the stability boundary and the postbuckling response of the plate. Sensitivity derivatives are evaluated and used to study the sensitivity of the postbuckling response to variations in the different lamination and material parameters of the plate. For quasi-isotropic plates, numerical results are presented showing the effects of variations in the material characteristics and fiber orientation of individual layers on the postbuckling response of the plate.

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THERMAL POSTBUCKLING OF THIN-WALLED COMPOSITE STIFFENERS

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(*Received* 5 *May* 1991)

Abstract—A study is made of the thermal postbuckling response of composite stiffeners subjected to prescribed edge displacement and a temperature rise. The flanges and web of the stiffeners are modeled by using two-dimensional plate finite elements. A mixed formulation is used with the fundamental unknowns consisting of the generalized displacements and the stress resultants of the plate. A reduction method is used in conjunction with mixed finite element models for determining the postbuckling response of the stiffeners. Sensitivity derivatives are evaluated and used to study the effects of variations in the different lamination and material parameters of the stiffeners on their postbuckling response characteristics. Numerical studies are presented for anisotropic stiffeners with Zee and channel sections.

Three-Dimensional Solutions for the Free Vibrations and Buckling of Thermally Stressed Multilayered Angle-Ply Composite Plates

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Analytic three-dimensional elasticity solutions are presented for the free vibration and buckling of thermally stressed, multilayered, angle-ply composite plates. Sensitivity derivatives are also evaluated and used to study the sensitivity of the vibration and buckling responses to variations in the different lamination and material parameters of the plate. The plates are assumed to have rectangular geometry and an antisymmetric lamination with respect to the middle plane. The temperature is assumed to be independent of the surface coordinates, but has an arbitrary symmetric variation through the thickness of the plate. A linear, Duhamel-Neumann type constitutive model is used, and the material properties are assumed to be independent of temperature. The thermal plate response is subjected to time-varying perturbation displacements, strains, and stresses. A mixed formulation is used with the fundamental unknowns consisting of the six perturbation stress components and the three perturbation displacement components of the plate. The initial thermal deformations are accounted for. Each of the plate variables is decomposed into symmetric and antisymmetric components in the thickness direction, and is expressed in terms of a double Fourier series in the Cartesian surface coordinates. Numerical results are presented showing the effects of variations in material characteristics and fiber orientation of different layers, as well as the effects of initial thermal deformations on the vibrational and buckling responses of the plate, as well as their sensitivity derivatives.

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AIAA JOURNAL
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Accuracy of Critical-Temperature Sensitivity Coefficients Predicted by Multilayered Composite Plate Theories

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An assessment is made of the accuracy of the critical-temperature sensitivity coefficients of multilayered plates predicted by different modeling approaches, based on two-dimensional shear-deformation theories. The sensitivity coefficients considered measure the sensitivity of the critical temperatures to variations in different lamination and material parameters of the plate. The standard of comparison is taken to be the sensitivity coefficients obtained by the three-dimensional theory of thermoelasticity. Numerical studies are presented showing the effects of variation in the geometric and lamination parameters of the plate on the accuracy of both the sensitivity coefficients and the critical temperatures predicted by the different modeling approaches.

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Reduced Basis Technique for Calculating Sensitivity Coefficients of Nonlinear Structural Response

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An efficient reduced basis technique is presented for calculating the sensitivity of nonlinear structural response to variations in the design variables. The structure is discretized by using two-field mixed finite element models. The vector of structural response and its sensitivity coefficients (derivatives with respect to design variables) are each expressed as a linear combination of a small number of basis (or global approximation) vectors. The Bubnov-Galerkin technique is then used to approximate each of the finite element equations governing the response and the sensitivity coefficients by a small number of algebraic equations in the amplitudes of these vectors. The path derivatives (derivatives of the response vector with respect to path parameters; e.g., load parameters) are used as basis vectors for approximating the response. A combination of the path derivatives and their derivatives with respect to the design variables is used for approximating the sensitivity coefficients. The potential of the proposed technique is discussed and its effectiveness is demonstrated by means of numerical examples of laminated composite plates subjected to mechanical and thermal loads.

Three-Dimensional Solutions for the Thermal Buckling and Sensitivity Derivatives of Temperature-Sensitive Multilayered Angle-Ply Plates

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Analytic three-dimensional thermoelasticity solutions are presented for the thermal buckling of multilayered angle-ply composite plates with temperature-dependent thermoelastic properties. Both the critical temperatures and the sensitivity derivatives are computed. The sensitivity derivatives measure the sensitivity of the buckling response to variations in the different lamination and material parameters of the plate. The plates are assumed to have rectangular geometry and an antisymmetric lamination with respect to the middle plane. The temperature is assumed to be independent of the surface coordinates, but has an arbitrary symmetric variation through the thickness of the plate. The prebuckling deformations are accounted for. Numerical results are presented, for plates subjected to uniform temperature increase, showing the effects of temperature-dependent material properties on the prebuckling stresses, critical temperatures, and their sensitivity derivatives.

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MECHANICS OF ANISOTROPIC PLATES AND SHELLS—A NEW LOOK AT AN OLD SUBJECT

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(*Received* 29 *April* 1992)

Abstract—A number of aspects of the mechanics of anisotropic plates and shells are discussed. The topics covered include computational models of anisotropic plates and shells, consequences of anisotropy on deformation couplings, symmetry types, stress concentrations and edge effects, and importance of transverse shear deformation, recent applications and recent advances in the modeling and analysis of anisotropic plates and shells; and new research directions.

Computational models for high-temperature multilayered composite plates and shells

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The focus of this review is on the hierarchy of composite models, predictor-corrector procedures, the effect of temperature-dependence of material properties on the response, and the sensitivity of the thermomechanical response to variations in material parameters. The literature reviewed is devoted to the following eight application areas: Heat transfer; thermal stresses; curing, processing and residual stresses; bifurcation buckling; vibrations of heated plates and shells; large deflection and postbuckling problems; and sandwich plates and shells. Extensive numerical results are presented showing the effects of variation in the lamination and geometric parameters of temperature-sensitive angle-ply composite plates on the accuracy of thermal buckling response, and the sensitivity derivatives predicted by nine different modeling approaches (based on two-dimensional theories). The standard of comparison is taken to be the exact three-dimensional thermoelasticity solutions. Some future directions for research on the modeling of high-temperature multilayered composites are outlined.

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Hierarchical adaptive modeling of structural sandwiches and multilayered composite panels

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•••

Abstract

Some recent advances in the hierarchical modeling strategies are reviewed with special emphasis on applications to multilayered composite panels. Discussion focuses on the key elements of hierarchical adaptive modeling; multimodel predictor–corrector modeling procedures; potential for solving large complex problems; and the needed development to realize this potential. Numerical studies are presented for free vibrations of thermally-stressed multilayered composite panels and structural sandwiches with composite face sheets demonstrating the effectiveness of the multimodel predictor–corrector modeling approaches.

Key words: ••• add: Hierarchical modeling, adaptivity, predictor-corrector procedures, finite elements, plates and shells, thermal buckling, free vibrations, sandwich structures, composites

FINITE ELEMENT BUCKLING AND POSTBUCKLING ANALYSES

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Abstract

An overview is given of the finite element buckling and postbuckling analyses of composite plates. Discussion focuses on finite element formulations of the prebuckling, bifurcation buckling and postbuckling responses; sensitivity analysis; application of reduction methods and other recent computational enhancements. Reported applications of the finite element method to buckling and postbuckling problems of composite plates are reviewed. Finite element solutions are presented for the buckling and postbuckling responses of composite plates subjected to various combinations of mechanical and thermal loadings. Some future directions for research are outlined.

To appear in "Buckling and Postbuckling of Composite Plates," edited by G. J. Turvey and I. H. Marshall, Elsevier (to appear).

FINITE ELEMENT BUCKLING AND POSTBUCKLING SOLUTIONS FOR MULTILAYERED COMPOSITE PANELS

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ABSTRACT

A study is made of the buckling and postbuckling responses of flat, unstiffened composite panels subjected to various combinations of mechanical and thermal loads. The analysis is based on a first-order shear deformation von-Karman type plate theory. A mixed formulation is used with the fundamental unknowns consisting of the strain components, stress resultants and the generalized displacements of the plate. The stability boundary, postbuckling response and the sensitivity coefficients are evaluated. The sensitivity coefficients measure the sensitivity of the buckling and postbuckling responses to variations in the different lamination and material parameters of the panel. Numerical results are presented for both solid panels and panels with central circular cutouts. The results show the effects of the variations in the fiber orientation angles, aspect ratio of the panel, and the hole diameter (for panels with cutouts) on the stability boundary, postbuckling response and sensitivity coefficients.

To appear in Finite Elements in Analysis and Design

TRANSVERSE SHEAR STRESSES AND THEIR SENSITIVITY COEFFICIENTS IN MULTILAYERED COMPOSITE PANELS

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ABSTRACT

A computational procedure is presented for the accurate determination of transverse shear stresses and their sensitivity coefficients in flat multilayered composite panels subjected to mechanical and thermal loads. The sensitivity coefficients measure the sensitivity of the transverse shear stresses to variations in the different lamination and material parameters of the panel. The panel is discretized by using either a three-field mixed finite element model based on a two-dimensional first-order shear deformation plate theory, or a two-field degenerate solid element with each of the displacement components having a linear variation throughout the thickness of the laminate. The evaluation of transverse shear stresses can be conveniently divided into two phases. The first phase consists of using a superconvergent recovery technique for evaluating the in-plane stresses in the different layers. In the second phase, the transverse shear stresses are evaluated by using piecewise integration, in the thickness direction, of the three-dimensional equilibrium equations. The same procedure is used for evaluating the sensitivity coefficients of the transverse shear stresses.

The effectiveness of the computational procedure is demonstrated by means of numerical examples of multilayered cross-ply panels subjected to transverse loading, uniform temperature change, and uniform temperature gradient through the thickness of the panel. In each case the standard of comparison is taken to be the exact solution of the three-dimensional thermoelasticity equations of the panel.

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